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THE STABILITY OF A BEAM SUBJECTED
TO A MOVING MASS

G. L. Anderson

February 1977



WATERVLIET ARSENAL WATERVLIET, N.Y. 12189

# TECHNICAL REPORT

AMCMS No. 611102.11.85300 DA Project No. 1T161102B14A Pron No. A1-5-51700



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| US Army Armament Command   |  | [77] February 1977   |
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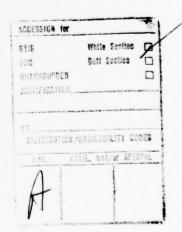
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## 1. INTRODUCTION

In two previous reports, Simkins et al [1], [2] derived the equation of motion of a simply supported, thin elastic beam subjected to an internal pressure (sometimes called a Bourdon force) and a concentrated mass moving at uniform velocity along its axis. The forced dynamic response of the system was determined with the aid of the finite element method. It was pointed out in these reports that if the moving mass is an appreciable fraction of the total mass of the structure or if the velocity of the mass approaches a critical value, the dynamic deflections of the system can become significantly large. In addition, if the moving mass remains on the beam for a sufficiently long period of time, the question of the stability of the system must be examined.

It is the objective of this report, then, to determine the fundamental zone of instability in the mass-velocity plane. Because of the complexity of the partial differential equation of motion, one cannot hope to determine easily the exact solution of the problem. However, through use of the Galerkin procedure, it is possible to obtain a useful approximate solution of the problem.

#### 2. THE EQUATION OF MOTION

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The partial differential equation of motion of a slender beam of length  $\ell$ , uniform cross sectional area A, and mass density  $\rho$  subjected to a gravity body force and a concentrated mass m moving at uniform velocity along its axis (the  $x_1$  - axis) is

EI
$$\omega$$
, | | | | + m( $\omega$ , t<sub>1</sub>t<sub>1</sub> + 2V $\omega$ , | | | + V<sup>2</sup> $\omega$ , | | + g)  $\delta(x_1 - Vt_1) + \rho A\omega$ , t<sub>1</sub>t<sub>1</sub> =  $\rho gA$ ,  $\sigma < x_1 < \ell$ , (1)

where  $\omega_{1} = \partial \omega/\partial x_{1}$ ,  $\omega_{1} = \partial \omega/\partial t_{1}$  ( $t_{1}$  denotes time), EI is the flexural rigidity of the beam, g the acceleration of gravity, and  $\delta(x_{1} - Vt_{1})$  the Dirac delta function. Since the beam is simply supported at both ends, the apposite boundary conditions are

$$\omega = \omega_{11} = 0 \text{ at } x_1 = 0.2$$
 (2)

The coefficient of the Dirac delta function in equation (1) represents the influence of the concentrated mass m moving along the  $x_1$ -axis. The objective of the analysis to follow consists of determining the values of the mass m and velocity V parameters for which the system becomes unstable. The term representing the effect of a uniform internal pressure has not been included in equation (1). Some aspects of the boundary value problem stated above have been discussed by Bolotin [3] and Inglis [4]. An interesting related problem has been solved by Stanisic and Hardin [5]. Fryba [6] has published a book that summarized the principal results of a fairly broad class of moving load problems.

If the changes of variables

$$\omega(x_1,t_1) = \ell y(x,t)$$
  $x_1 = \ell x$   $t_1 = at$ 

are made and the parameters

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$$a^2 = \rho A \ell^4 / EI$$
,  $\mu = m/\rho A \ell$ ,  $M = \rho g A \ell^3 / EI$ ,  $v = V a / \ell$ , (3)

are introduced, then the boundary value problem in equations (1) and (2) can be expressed in dimensionless form as

$$y'' + \mu(M + \ddot{y} + 2v\dot{y}' + v^2y'') \delta(x - vt) + \ddot{y} - M = 0, o < x < 1,$$
 (4)

$$y = y'' = 0$$
 at  $x = 0,1,$  (5)

where  $y' = \partial y/\partial x$ ,  $\dot{y} = \partial y/\partial t$ , etc.

## 3. THE GALERKIN PROCEDURE

In view of the boundary conditions in equation (5), one is motivated to assume a solution for equation (4) in the form

$$y(x,t) = \sum_{n=1}^{\infty} q_n(t) \sin(n\pi x), \qquad (6)$$

where the generalized coordinates  $q_n$  are unknown functions of time. Substitution of equation (6) into equation (4), multiplication of the result by  $\sin(k\pi x)$ , and integration of the product yield

$$\ddot{q}_{k} + (k\pi)^{4} q_{k} + 2\mu \sum_{n=1}^{N} [\ddot{q}_{n} \sin n\theta + 2n\pi\nu\dot{q}_{n} \cos n\theta - (n\pi\nu)^{2} q_{n} \sin n\theta] \sin k\theta = \frac{2M}{k\pi} [1-(-1)^{k}] - 2\mu M \sin k\theta, \qquad (7)$$

in which  $\ddot{q}_k = d^2q_k/dt^2$  and  $\theta = \pi vt$ .

The completed solution of equation (7) can be expressed as

$$q_n(t) = q_n^c(t) + q_n^p(t),$$

which simply represents a superposition of the complementary and the particular solutions. For the purpose of determining the state of stability of the system under consideration, it is sufficient to examine the complementary solution only which satisfies the ordinary differential equation

$$\ddot{q}_{k} + (k\pi)^{4} q_{k} + 2\mu \sum_{n=1}^{N} [\ddot{q}_{n} \sin n\theta + 2n\pi v \dot{q}_{n} \cos n\theta - (n\pi v)^{2} q_{n} \sin n\theta] \sin k\theta = 0,$$
 (8)

where the superscript c has been deleted.

If N = 1, equation (8) becomes simply

$$\ddot{q}_1 + \pi^4 q_1 + \mu [q_1(1 - \cos 2\theta) + 2\pi v \dot{q}_1 \sin 2\theta - (\pi v)^2 q_1 (1 - \cos 2\theta)] = 0.$$
 (9)

For N = 2, one has a pair of coupled differential equations, namely,

$$\ddot{q}_1 + \pi^4 q_1 + \mu [\ddot{q}_1(1 - \cos 2\theta) + 2 v\dot{q}_1 \sin 2\theta - (\pi v)^2 q_1 (1 - \cos 2\theta) + q_2 (\cos \theta - \cos 3\theta) +$$

+ 
$$4\pi v \dot{q}_2 (\sin 3\theta - \sin \theta) - (2\pi v)^2 q_2 (\cos \theta - \cos 3\theta)] = 0$$
 (10)

and  $\ddot{q}_2 + (2\pi)^4 q_2 + \mu [\ddot{q}_1(\cos\theta - \cos 3\theta) + 2\pi v \dot{q}_1(\sin\theta + \sin 3\theta) - (\pi v)^2 q_1(\cos\theta - \cos 3\theta) + \ddot{q}_2(1 - \cos 4\theta) +$ 

+ 
$$4\pi v \dot{q}_2 \sin 4\theta - (2\pi v)^2 q_2 (1 - \cos 4\theta)] = 0.$$
 (11)

#### 4. THE STABILITY BOUNDARIES

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A useful approximation for the stability boundaries in the  $\nu_{\mu}$ plane can be obtained from the solution of equation (9). If more
refined results might be required, one must solve equations (10) and
(11), to which the methods of references [7] and [8] probably may be applied.

If one defines  $\omega = \pi v$ ,  $\theta = \omega t$ ,  $\overline{\gamma} = \pi^2/\omega$ , and  $q_1(t) = q(\theta)$ , then equation (9) can be expressed as

$$\frac{d^2q}{d\theta^2} + \overline{\gamma}^2 q + \mu \left[ \frac{d^2q}{d\theta^2} (1 - \cos 2) + \frac{2dq}{d\theta} \sin 2 - q(1 - \cos 2) \right] = 0, (12)$$

which is related to a Mathieu or Hill type of ordinary differential equation with periodic coefficients. To solve equation (12), it is efficacious to use Whittaker's technique [9] which was first used for the purpose of solving Mathieu's equation. This technique exploits the fact that equation (12) has a solution of the form

$$q(\theta) = e^{r\theta} \phi(\theta),$$
 (13)

where  $\phi(\theta)$  is a periodic function. Therefore, the state of stability of the system can be ascertained once the coefficient r in the exponential factor of the solution in equation (13) is determined as a function of the parameters of the system. In particular, under the assumption that r is real, the motion consists of (i) oscillations with an exponentially decaying amplitude (asymptotic stability) if r<0, (ii) oscillations of constant amplitude (neutral stability) if r=0, and (iii) oscillations with exponentially growing amplitude (instability) if r>0. Thus, the boundaries of regions of stability

in stability maps must be obtained from the condition

$$r = 0.$$
 (14)

Substitution of equation (13) into equation (12) yields  $\phi'' + 2r\phi' + (r^2 + \overline{\gamma}^2)\phi + \mu[(\phi'' + 2r\phi' + r^2\phi)(1 - \cos 2\theta) +$ 

$$+ 2(r\phi + \phi') \sin 2\theta - (1 - \cos 2\theta)] = 0,$$
 (15)

where  $\phi' = d\phi/d\theta$ , etc. If the mass ratio  $\mu$ , as defined in equation (3), is a small number relative to unity, a regular perturbation procedure, employing  $\mu$  as the expansion parameter, can be applied to equation (15). Of course,  $\mu$ <1 whenever the mass of the moving particle is less than the mass of the beam. If  $\mu$ >1, then it may be possible to use a perturbation method upon introducing a new small parameter, say,  $\epsilon$  defined as  $\epsilon = \mu/(1 + \mu)$ , where o< $\epsilon$ <1. Such a scheme has been used in reference [10]. Throughout the remainder of this report, it will be assumed that  $\mu$ <1.

Hence, one may suppose that

$$\omega^2 = \alpha^2 [1 + \mu \beta_1 + \mu^2 \beta_2 + (\mu^3)], \qquad (16)$$

where  $\alpha,~\beta_1,~\beta_2,~\dots$  are unknowns, so that

$$\overline{\gamma}^{2} = \pi^{4}/\omega^{2} = (\pi^{2}/\alpha)^{2} [1 + \mu\beta_{1} + \mu^{2}\beta_{2} + (\mu^{3})]^{-1}$$

$$= \gamma^{2} [1 - \mu\beta_{1} + \mu^{2}(\beta_{1}^{2} - \beta_{2}) + (\mu^{3})],$$
(17)

with  $\gamma = \pi^2/\alpha$ . Furthermore, it is assumed that

$$r = \mu r_1 + \mu^2 r_2 + (\mu^3),$$
 (18)

$$\phi(\theta) = \sin (\gamma \theta - \theta_0) + \mu \phi_1(\theta) + \mu^2 \phi_2(\theta) + (\mu^3),$$
 (19)

where  $\theta_0$  is an arbitrary phase angle. This set of formulas yields the exact expression for  $q(\theta)$  when  $\mu = 0$ .

Substitution of equations (17) to (19) into equations (15) and utilization of the familiar steps of the classical regular perturbation procedure, one obtains, writing  $\psi = \gamma\theta - \theta_0$ ,

$$\phi_{1}^{"} + \gamma^{2}\phi_{1} = -2r_{1}\gamma\cos\psi + (1 + \gamma^{2} + \gamma^{2}\beta_{1})\sin\psi - \frac{1}{2}(1 + \gamma)^{2}\sin(\psi + 2\theta) - \frac{1}{2}(1 - \gamma)^{2}\sin(\psi - 2\theta), \qquad (20)$$

$$\phi_{2}^{"} + \gamma^{2}\phi_{2} = -r_{1}\sin\psi - 2r_{2}\gamma\cos\psi - 2r_{1}\phi_{1}^{'} + \gamma^{2}\beta_{1}\phi_{1} - \frac{1}{2}(\beta_{1}^{2} - \beta_{2})\sin\psi - (\phi_{1}^{"} + 2r_{1}\gamma\cos\psi)(1 - \cos2\theta) - \frac{1}{2}(\phi_{1}^{'} + r_{1}\sin\psi)\sin2\theta + \phi_{1}(1 - \cos2\theta), \qquad (21)$$

etc. Equations (20) and (21) are non-homogeneous ordinary differential equations that can be solved by elementary methods.

The arguments of the trigonometric terms on the right side of equation (20) are

$$\psi+2\theta=(\gamma+2)\theta-\theta_0 \quad \text{and} \quad \psi-2\theta=(\gamma-2)\theta-\theta_0.$$
 If, in particular,  $\gamma-2=-\gamma,$  then  $\gamma=1,$  which implies that  $\alpha=\pi^2,$  and

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$$\psi + 2\theta = 3\theta - \theta_0$$
 and  $\psi - 2\theta = -(\theta + \theta_0)$ . (22)

Substitution of equation (22) into equation (20) yields, in this circumstance,

$$\phi_{1}^{"} + \phi_{1} = -2r_{1} \cos (\theta - \theta_{0}) + (2 + \beta_{1}) \sin (\theta - \theta_{0})$$

$$-2 \sin (3\theta - \theta_{0}). \qquad (23)$$

To avoid secular terms in  $\phi_1(\theta)$ , one must equate the coefficients of  $\cos(\theta-\theta_0)$  and  $\sin(\theta-\theta_0)$  to zero. Consequently,

$$r_1 = 0, \quad \beta_1 = -2,$$
 (24)

and

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$$\phi_1'' + \phi_1 = -2 \sin (3\theta - \theta_0).$$
 (25)

The particular solution of equation (25) is

$$\phi_1(\theta) = \frac{1}{4} \sin (3\theta - \theta_0).$$
 (26)

Substitution of  $\gamma$  = 1 and equation (26) into equation (21) yields, after some rearrangement,

$$\phi_{2}^{"} + \phi_{2} = -(2r_{2} - \frac{3}{8}\sin 2\theta_{0})\cos (\theta - \theta_{0}) + (\beta_{2} - \frac{21}{4} + \frac{3}{4}\cos 2\theta_{0})\sin (\theta - \theta_{0}) + 2\sin (3\theta - \theta_{0}) - 2\sin (5\theta - \theta_{0}).$$
 (27)

To eliminate secular terms in  $\phi_2(\theta)$ , one sets the coefficients of cos  $(\theta - \theta_0)$  and sin  $(\theta - \theta_0)$  in equation (27) equal to zero. Hence,

$$r_2 = \frac{3}{8} \sin 2\theta_0, \quad \beta_2 = \frac{21}{4} - \frac{3}{4} \cos 2\theta_0, \quad (28)$$

and

$$\phi_2'' + \phi_2 = 2 \sin (3\theta - \theta_0) - 2 \sin (5\theta - \theta_0).$$

By virtue of equations (24) and (28), one has from equation (18)

$$r = \frac{3}{8} \mu^2 \sin 2\theta_0 + (\mu^3).$$
 (29)

Therefore, it follows immediately from equations (14) and (29) that the condition for neutral stability is  $\sin 2\theta = 0$ , whence  $\theta = 0$  and  $\pi/2$ . Hence, by the second relation in equation (28),

$$\beta_2 = \frac{1}{4} [21 - 3 \cos 2\theta_0]_{\theta_0} = 0, \ \pi/2 = \begin{cases} 9/2 \text{ if } \theta_0 = 0, \\ 6 \text{ if } \theta_0 = \pi/2. \end{cases}$$
 (30)

Finally, from  $\omega = \pi \mathbf{v}$ ,  $\alpha = \pi^2$ , and equations (16), (24), and (30), one determines

$$v^2 = \pi^2 \left[1 + \mu \beta_1 + \mu^2 \beta_2 + (\mu^3)\right]_{\theta_0} = 0, \pi/2,$$

whence the expressions for the stability boundaries are found to be

$$v = \pi(1 - 2\mu + 9\mu^2/2)^{1/2}, \qquad (31)$$

and

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$$v = \pi (1 - 2\mu + 6\mu^2)^{1/2}. \tag{32}$$

Equations (31) and (32) provide the boundary curves for the shaded zone of instability (i.e., oscillations with an exponentially growing amplitude) shown in the stability map in the  $\nu_{\mu}$ -plane in Figure 1. Whenever the values of the parameters  $\nu$  and  $\mu$  are such that the point  $(\nu/\pi,\mu)$  falls within this shaded region, one must conclude that the motion of the system is unstable. Whenever the point  $(\nu/\pi,\mu)$ 

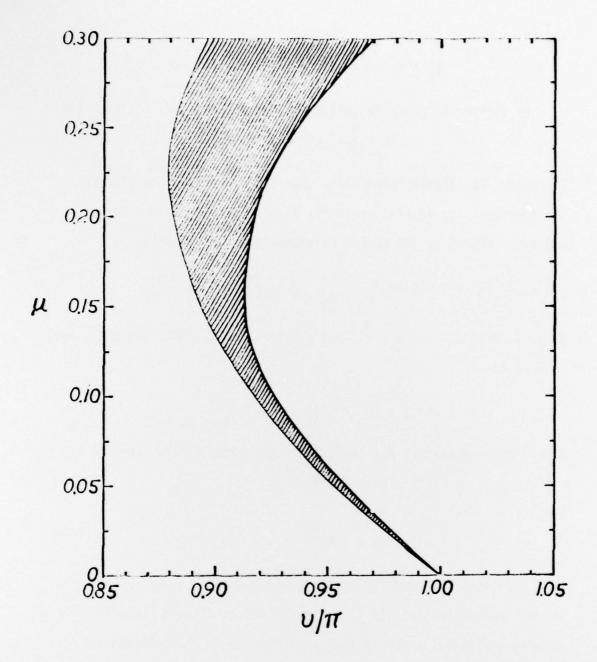


Figure 1. Stability map in the  $\nu\mu$ -plane.

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falls within the unshaded region, the motion of the system is asymptotically stable. Clearly, the region of instability appears at  $\mathbf{v}=\pi$  when  $\mu=0$ . As the value of  $\mu$  is increased, the values of  $\mathbf{v}$  for instability decrease initially below  $\mathbf{v}=\pi$ , but, as  $\mu$  becomes larger, the width of this region grows rapidly.

Without determining additional terms for the radicands in equations (31) and (32), one can easily derive more accurate approximations to the sums of the implied series than the original three terms, which are of the form

$$(v/\pi)^2 = 1 - a\mu + b\mu^2 + \dots,$$
 (33)

where a = 2 and b = 9/2 or 6 for equation (31) or (32), respectively. To accomplish this, let

$$S_1 = 1$$
,  $S_2 = 1 - a\mu$ ,  $S_3 = 1 - a\mu + b\mu^2$ 

denote the first three partial sums of the infinite series in equation (33). Shanks [11] has shown that the ngn-linear transformation

$$(v/\pi)^2 = \frac{S_3S_1^-S_2^2}{S_1^+S_3^-2S_2} = \frac{a+(b-a^2)\mu}{a+b\mu}$$

yields a more accurate value of the sum of the series in equation (33). In particular, one may replace equations (31) and (32) by

$$v = \pi[(4 + \mu)/(4 + 9\mu)]^{1/2},$$
 (34)

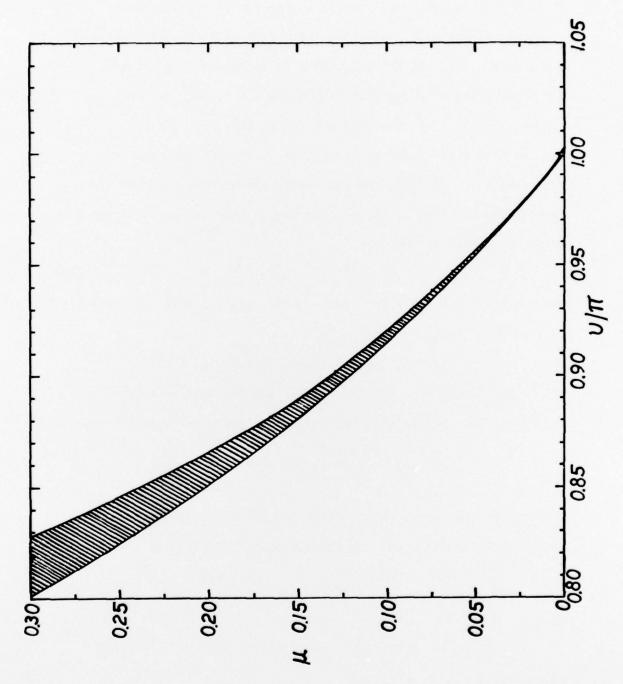
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$$v = \pi[(1 + \mu)/(1 + 3\mu)]^{1/2}$$

respectively.

In Figure 2, a stability map that is more accurate than the one



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given in Figure 1 is plotted. The boundary curves for the shaded zone of instability are those obtained from equations (34) and (35). Actually, a close comparison of Figures 1 and 2 reveals that the portions of the stability boundaries obtained from equations (31) - (32) and (34) - (35) are in excellent agreement for  $0 \le \mu < 0.1$ . For  $\mu > 0.1$ , however, the curves in Figure 2 do not show the marked reverse curvature that is evident in Figure 1. Moreover, the width of the zone of instability in Figure 2 does not increase so rapidly as that shown in Figure 1.

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